

**RECENT EXPERIENCE IN
SIMULTANEOUS CONTROL-STRUCTURE OPTIMIZATION**

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MOTIVATIONS: The optimization of a structure and its control system has traditionally proceeded along two separate but sequential paths. First, the structure is optimized by selecting a set of member sizes \underline{a}^* which minimize a structural criterion $J_s(\underline{a})$, subject to constraints $h_s(\underline{a})$, Eq (1). Then having specified the optimal structure, one may use the control theory to determine an optimal set of control variables \underline{u}^* that optimize a control criterion J_c subject to constraints $h_c(\underline{u})$, Eq. (2). This two-step optimization procedure is the so-called separate optimization and is equivalent to finding the linear sum of two separate minima, Eq. (3). The question arises then as to whether it is possible to achieve a superior combined optimum $(\underline{a}^{**}, \underline{u}^{**})$ over $(\underline{a}^*, \underline{u}^*)$ had one combined the two problems before, Eq. (4), rather than after, Eq. (3), the minimization. Intuitively, the answer to this question is affirmative, Eq. (5), since the minimum of the sum is less than the sum of the minima.

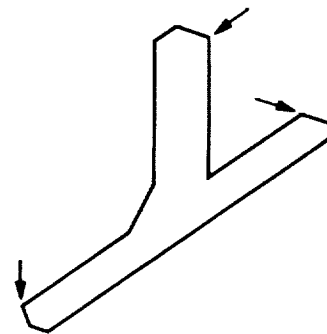
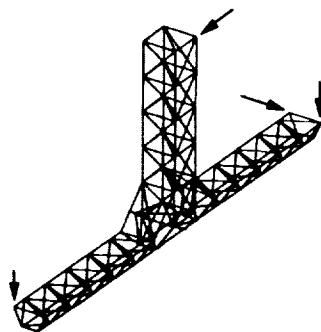
• SEPARATE OPTIMIZATION

$$J_s(\underline{a}^*) = \min_{\underline{a}} J_s(\underline{a}) \quad (1)$$

$$h_s(\underline{a}) \geq 0, \underline{a}^* \in \underline{a}$$

$$J_c(\underline{a}^*, \underline{u}^*) = \min_{\underline{u}} J_c(\underline{a}^*, \underline{u}) \quad (2)$$

$$h_c(\underline{u}) \geq 0, \underline{u}^* \in \underline{u}$$



$$J(\underline{a}^*, \underline{u}^*) = J_s(\underline{a}^*) + J_c(\underline{a}^*, \underline{u}^*) = \min_{\underline{a}} J_s + \min_{\underline{u}} J_c \quad (3)$$

• SIMULTANEOUS OPTIMIZATION

$$J(\underline{a}^{**}, \underline{u}^{**}) = \min_{\underline{a}, \underline{u}} [J_s(\underline{a}) + J_c(\underline{a}, \underline{u})] \quad (4)$$

$$J(\underline{a}^{**}, \underline{u}^{**}) \leq J(\underline{a}^*, \underline{u}^*) \quad (5)$$

DEFINITIONS: A common starting point for most approaches begins with the second order dynamical equation, Eq. (6), in n_s degrees of freedom (d.o.f.). The \underline{M} , \underline{D} , \underline{K} matrices are the mass, damping, and stiffness. \underline{G}_1 = the disturbance influence matrix, \underline{G}_2 = control influence matrix, \underline{a} = structure design variables, \underline{u} = control variables, \underline{w} = disturbance vector, and \underline{v} = physical d.o.f.

Let $\underline{x} = (\underline{v}, \dot{\underline{v}})^T$ = state variables, then the equation of state is given by (7), with output consisting of controlled states \underline{z} and measured states \underline{y} , both related to \underline{x} by Eq. (8). Where \underline{A} , \underline{B}_1 , \underline{B}_2 , \underline{C}_1 and \underline{C}_2 are defined by Eq. (9).

$$\underline{M}(\underline{a}) \ddot{\underline{v}} + \underline{D}(\underline{a}) \dot{\underline{v}} + \underline{K}(\underline{a}) \underline{v} = \underline{G}_1 \underline{w} + \underline{G}_2 \underline{u} \quad (6)$$

$$\underline{x} = (\underline{v}, \dot{\underline{v}})^T$$

$$\dot{\underline{x}} = \underline{A}(\underline{a}) \underline{x} + \underline{B}_1(\underline{a}) \underline{w} + \underline{B}_2(\underline{a}) \underline{u} \quad (7)$$

$$\text{CONTROLLED STATES} \quad \underline{z} = \underline{C}_1 \underline{x} \quad (8)$$

$$\text{MEASURED STATES} \quad \underline{y} = \underline{C}_2 \underline{x}$$

$$\underline{A} = \left(\begin{array}{c|c} \underline{0} & \underline{I} \\ \hline -\underline{M}^{-1}\underline{K} & -\underline{M}^{-1}\underline{D} \end{array} \right); \underline{B}_1 = \left(\begin{array}{c} \underline{0} \\ -\underline{M}^{-1}\underline{G}_1 \end{array} \right); \underline{B}_2 = \left(\begin{array}{c} \underline{0} \\ -\underline{M}^{-1}\underline{G}_2 \end{array} \right)$$

$$\underline{C}_1 = (C_{11}, C_{12}); \underline{C}_2 = (C_{21}, C_{22}) \quad (9)$$

FORMULATION: Herein, we focus on LQ-based formulation as a natural one to generalize to the simultaneous control-structure optimization. Two types of controllers are considered; state feedback and output feedback. For both of these, the control criterion J_c is taken as a quadratic function of the structural response and control energy, Eq. (10). For the structural criterion, we assume one that depends only on the structural variables \underline{a} . As will become clear later, this simplifies the derivations considerably. An example of such structural criteria is the mass of the structure, $M(\underline{a})$.

The simultaneous optimization problem consists of finding the structure and control variables (\underline{a}^{**} , \underline{u}^{**}) that minimize the combined criterion (11), subject to any behavioral constraints (12), and/or side constraints (13) providing upper and/or lower bounds on the design variables (\underline{a} , \underline{u}). Since the terms in (11) do not have the same units, the scalar α and matrices \underline{Q} and \underline{R} can be chosen on computational and physical grounds.

- STATE FEEDBACK
- OUTPUT FEEDBACK
- INDIVIDUAL CRITERIA:

$$\begin{aligned} \text{CONTROL:} \quad J_c(\underline{a}, \underline{u}) &= \int_0^\infty (\underline{x}^T \underline{Q} \underline{x} + \underline{u}^T \underline{R} \underline{u}) dt \\ \text{STRUCTURE:} \quad J_s(\underline{a}) &= M(\underline{a}) \end{aligned} \quad (10)$$

- COMBINED CRITERION: FIND (\underline{a}^{**} , \underline{u}^{**}), $\underline{a}^{**} \in \underline{a}$, $\underline{u}^{**} \in \underline{u}$

$$J(\underline{a}, \underline{u}) = \min_{\underline{a}, \underline{u}} [\alpha M + \int_0^\infty (\underline{x}^T \underline{Q} \underline{x} + \underline{u}^T \underline{R} \underline{u}) dt] \quad (11)$$

$$\text{SUBJECT TO:} \quad h_j(\underline{a}, \underline{u}) \geq 0 \quad (12)$$

$$\underline{a} \leq \underline{a}_i \leq \bar{\underline{a}}, \quad \underline{u} \leq \underline{u}_i < \bar{\underline{u}} \quad (13)$$

STATE FEEDBACK: With the assumption that the structural objective M is dependent upon \underline{a} only, Eq. (11) simplifies to (14) and then to (15). This allows the familiar analytical solution for the optimal control u^{**} in (18) and its companion equations (16) and (17). The necessary conditions for the minimum of (15) subject to the constraint (17) and the constraints imposed by (12) can be derived by first forming an auxiliary Lagrangian function, then setting its partial derivatives at the local minimum to zero. This yields conditions (19), (20) and (21), from which the optimal \underline{a}^{**} can be computed iteratively. For a given \underline{a} , Eqs. (21) and (22A) are solved for P and $P_{,a}$. With these, Eq. (19) is solved for an updated \underline{a} , and (20) is evaluated to check the constraints.

$$\bullet J(a, u) = \min_{\underline{a}} [\alpha M(a) + \min_{\underline{u}} \int (\underline{x}^T \underline{Q} \underline{x} + \underline{u}^T \underline{R} \underline{u}) dt] \quad (14)$$

$$= \min_{\underline{a}} [\alpha M(a) + \text{Tr}(P Q_0)] \quad (15)$$

$$Q_0 = B_1 B_1^T \quad \text{IF DIST } W \text{ IS UNIT IMPULSE} \quad (16)$$

$$= \underline{x}_0 \underline{x}_0^T \quad \text{IF DIST IS INITIAL COND } \underline{x}(0) = \underline{x}_0$$

$$P \text{ SATISFIES } A^T P + P A + Q - P(B_2 R^{-1} B_2^T) P = 0 \quad (17)$$

$$u^{**} = -R^{-1} B_2^T P x \quad (18)$$

• CONDITIONS OF OPTIMALITY

$$\hat{J}_{a_i} = \alpha \tilde{M}_{,a_i} + \text{Tr}(P Q_{0,a_i} + P_{,a_i} Q_0) + \sum_j \ell_j h_{j,a_i} = 0 \quad (19)$$

$$\ell_j h_j = 0 \quad (20)$$

$$A^T P + P A + Q - P B_2 R^{-1} B_2^T P = 0 \quad (21)$$

$$A_c^T P_{,a_i} + P_{,a_i} A_c + [A_{,a_i}^T P + P A_{,a_i} - P(B_2 R^{-1} B_2^T)_{,a_i} P + Q_{,a_i}] = 0 \quad (22)A$$

$$A_c = A - B_2 R^{-1} B_2^T P$$

OUTPUT FEEDBACK: To avoid the state reconstruction necessary to implement the full state feedback control design, the static output feedback approach requires only the output of the measured states \underline{y} . With a controller of the form of Eq. (22B), in which the gain \underline{F} is assumed to stabilize the structure, the combined criterion in (11) reduces to (23). Equation (23) is similar to its counterpart, Eq. (15) for the state feedback, except now the output feedback gain \underline{F} may be considered as an optimization variable in addition to \underline{a} . Furthermore, $\underline{P}(\underline{a}, \underline{F}) = \underline{P}^T(\underline{a}, \underline{F}) > 0$ satisfies the Lyapunov Eq. (24).

Here again, the necessary conditions for the minimum of Eq. (23), subject to the constraints of Eqs. (24) and (12), can be found by forming the auxiliary Lagrangian function and setting its partial derivatives at the local minimum to zero. This leads to Eqs. (25) to (29), which must be solved iteratively. For a given $(\underline{a}, \underline{F})$, Eqs. (27), (28) and (29), respectively, allow the solution of \underline{P} , the Lagrangian matrix multiplier \underline{L} and the scalar Lagrangians $\ell_j \geq 0$ for each behavioral constraint h_j . With these, Eqs. (25) and (26) yield an improved $(\underline{a}, \underline{F})$, and so on.

- ASSUMES $\underline{u} = \underline{F}\underline{y}$ (F STABILIZES STRUCTURE) (22)B

- FIND MIN. OF $J(\underline{a}, \underline{u}) = \min_{\underline{a}, \underline{F}} [\alpha \underline{M} + \text{Tr}(\underline{P}\underline{Q}_0)]$ (23)

WHERE $\underline{P}(\underline{a}, \underline{F})$ SATISFIES $\underline{A}_c^T \underline{P} + \underline{P} \underline{A}_c + \underline{Q}_c = 0$ (24)

$$\underline{A}_c = \underline{A} + \underline{B}_2 \underline{F} \underline{C}_2$$

$$\underline{Q}_c = \underline{Q} + \underline{C}_2^T \underline{F}^T \underline{R} \underline{F} \underline{C}_2$$

- CONDITIONS OF OPTIMALITY

$$\alpha \underline{M}, \underline{a}_i + \text{Tr}[\underline{P}\underline{Q}_0, \underline{a}_i + 2\underline{L}\underline{P}\underline{A}_c, \underline{a}_i + \underline{L}\underline{Q}_c, \underline{a}_i] + \sum_j \ell_j h_j, \underline{a}_i = 0 \quad (25)$$

$$\text{Tr}[2\underline{L}\underline{P}\underline{A}_c, \underline{F} + \underline{L}\underline{Q}_c, \underline{F}] + \sum_j \ell_j h_j, \underline{F} = 0 \quad (26)$$

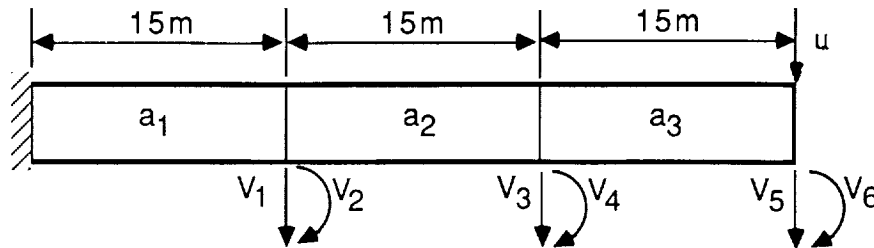
$$\underline{A}_c^T \underline{P} + \underline{P} \underline{A}_c + \underline{Q}_c = 0 \quad (27)$$

$$\underline{L} \underline{A}_c^T + \underline{A}_c \underline{L} + \underline{Q}_0 = 0 \quad (28)$$

$$\ell_j h_j = 0 \quad (29)$$

EXAMPLE 1: STATE FEEDBACK (Ref. 1)

The cantilever beam shown is modeled by three finite elements with cross-sectional areas $\underline{a} = (a_1, a_2, a_3)^T$, and has six d.o.f. An initial deformation vector at the six d.o.f. $x(0) = x_0$ is specified, and a control force u is applied at the free tip. The areas \underline{a} and control u are to be determined so as to minimize Eq. (11) while maintaining a fundamental open-loop frequency $\omega > 0.10$ rad/sec. Rather than a first order minimization, it was found necessary for faster convergence to use a second order scheme based on modified Newton-Raphson iterations. For this purpose, the design variables \underline{a} and multiplier ℓ are obtained iteratively from the recursive relations in Eq. (30).



$$\underline{x}_0 = (0.011, 0.00135, 0.037, 0.002, 0.0688, 0.00216)^T$$

$$\omega \geq 0.10 \text{ rad/sec. (i.e. } h = \omega^2 - (0.10)^2 \geq 0)$$

$$E = 9.56 \cdot 10^{10} \text{ N/m}^2, \rho = 1660 \text{ Kg/m}^3$$

DAMPING = 0.5% CRITICAL

$$\begin{Bmatrix} \underline{a} \\ \ell \end{Bmatrix}_{r+1} = \begin{Bmatrix} \underline{a} \\ \ell \end{Bmatrix}_r - S \left[\begin{array}{c|c} J, \underline{a} \underline{a} + \sum_j \ell_j h_{j, \underline{a}} \underline{a} & h_{j, \underline{a}} \\ \hline h_{j, \underline{a}} & 0 \end{array} \right]^{-1} \begin{Bmatrix} J, \underline{a} + \sum_j \ell_j h_{j, \underline{a}} \\ h_{j, \underline{a}} \end{Bmatrix}_r \quad (30)$$

ALGORITHM: The algorithm begins with a feasible initial design and a step length s with which a line search in the direction of negative gradient is performed. This is continued until the minimum is reached, or until the constraint is violated. If the latter occurs, constrained minimization is employed with an initial estimate of the multiplier λ from

$$\lambda = -[H^T H]^{-1} H^T J_{,a}$$

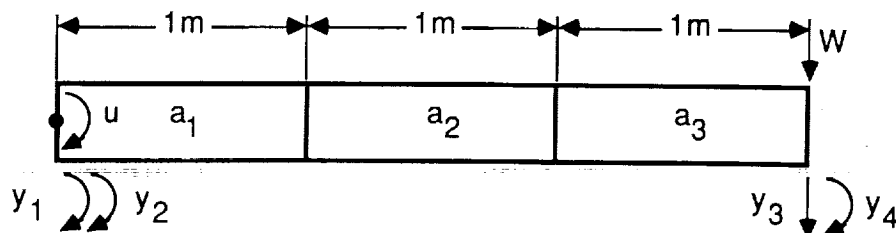
where

$$H = [h_{1,a} \dots h_{n,a}]$$

With this Eq. (30) is used. The constraint is checked continually. If the design moves away from the constraint, unconstrained minimization is reverted to. Thus the minimization process alternates between iterations which involve unconstrained minimization and iterations which involve constrained minimization as outlined below.

EXAMPLE 2: OUTPUT FEEDBACK (Ref. 2)

In this example, an active disturbance force is applied to the free end of the beam in the figure below. At the other end the beam is pinned and a control torque is applied there. The measurements consist of angular deformation and angular velocity at the free end. The design variables for minimization are the cross-sectional areas $\underline{a} = (a_1, a_2, a_3)^T$ and the gains $\underline{F} = (F_1, \dots, F_4)^T$. No behavioral constraints are imposed. Other parameters of the problem are listed below. Since there are no constraints, the minimization algorithm is essentially similar to the unconstrained gradient search portion of the algorithm described previously.

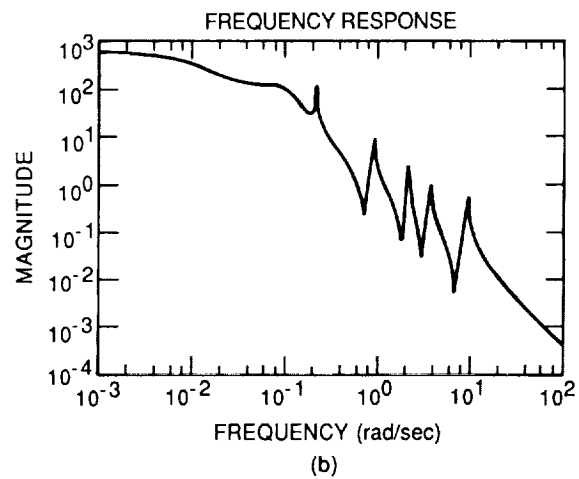
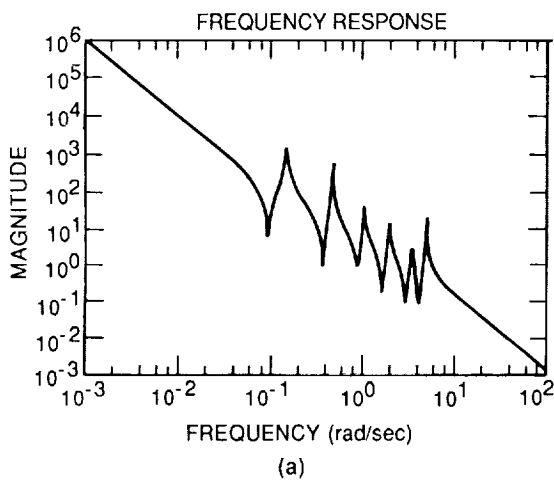


$$E = 800 \text{ N/m}^2, \rho = 100 \text{ Kg/m}^3$$

EXAMPLE 2 - RESULTS: The numerical results in the table below compare \underline{a} and \underline{F} , and the resultant mass and combined index J for the initial design and optimized design. A factor of three reduction in J is realized as a consequence of simultaneous optimization over \underline{a} and \underline{F} . In the accompanying plots, the transfer functions of the initial and optimum design from disturbance to the controlled output show three orders of magnitude reduction in response.

SIMULTANEOUS OPTIMIZATION

	INITIAL DESIGN				SIMULTANEOUS OPTIMIZATION			
AREAS	a_1	a_2	a_3		a_1	a_2	a_3	
	0.1	0.1	0.1		0.02308	0.01654	0.03572	
GAINS	F_1	F_2	F_3	F_4	F_1	F_2	F_3	F_4
	-1.0	-1.0	0.0	0.0	-0.0306	-1.2872	-1.2387	+0.1136
MASS	30.0				7.4			
\hat{J}_{\min}	3.06×10^4				9.20×10^3			



FREQUENCY RESPONSE OF (a) INITIAL DESIGN AND (b) OPTIMUM DESIGN

CONCLUDING REMARKS: To show the feasibility of simultaneous optimization as design procedure, we have used low-order problems in conjunction with simple control formulations. The numerical results indicate that simultaneous optimization is not only feasible - but also advantageous. Such advantages come at the expense of introducing complexities beyond those encountered in structure optimization alone, or control optimization alone. Examples include: larger design parameter space, optimization may combine continuous and combinatoric variables, and the combined objective function may be nonconvex.

Future extensions to include large order problems, more complex objective functions and constraints, and more sophisticated control formulations will require further research to ensure that the additional complexities do not outweigh the advantages of simultaneous optimization. Some areas requiring more efficient tools than currently available include: multiobjective criteria and nonconvex optimization. We also need to develop efficient techniques to deal with optimization over combinatoric and continuous variables, and with truncation issues for structure and control parameters of both the model space as well as the design space.

- SIMPLE FORMULATIONS USED WITH LOW-ORDER PROBLEMS
- RESULTS SHOW SIMULTANEOUS OPTIMIZATION FEASIBLE AND ADVANTAGEOUS
- ADDITIONAL COMPLEXITIES: - LARGER PARAMETER SPACE
 - POSSIBLE NONCONVEXITY OF OBJECTIVE FN.
 - MIXTURE OF CONTINUOUS AND COMBINATORIC VARIABLES
- FURTHER EXTENSIONS: - LARGER PROBLEMS
 - OTHER OBJECTIVE FNs, CONSTRAINTS, MORE SOPHISTICATED CONTROL FORMULATIONS
 - MORE EFFICIENT TOOLS TO DEAL WITH ABOVE COMPLEXITIES
 - UNIFIED TRUNCATION METHODOLOGY FOR CONTROL & STRUCTURE PARAMETERS OF MODEL SPACE & DESIGN SPACE

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